Summer Assignment and General Information.

Welcome to AP Calculus BC! I am looking forward to going on this adventure with you this coming school year.

For your summer assignment, I have attached a packet of cumulative review questions from the past few years of your math careers. You will see throughout the year how Calculus builds upon all of the skills you have acquired from your previous courses. This packet will be due on the first day of school (September 7, 2023). You must show all work, for all problems. If you need more space than provided to complete a problem, please do it on a separate sheet of paper, clearly label the problem and staple it to the back of the packet. If you are unable to print the packet, please copy all problems down in pen and solve underneath using pencil.

I expect you to have a binder for this class (not a shared notebook with another class, I recommend at least a 2 in), a graphing calculator, paper, writing utensils and a positive attitude with you at each class meeting- including day 1! You will get frustrated, but keep at it. This class is the epitome of problem solving. Talk things over with classmates, get a study group together; this is encouraged. Sometimes you may have to walk away from a problem, and then go back to it. Perseverance will be a key to success.

Tests will be cumulative throughout the year. If you are present the day of a test, you will be expected to take it that day. If you are not present the day of a test, you will be expected to take it the next day you are in class or at another time the same day depending on the nature of the absence.

Feel free to email with any questions over the summer and/or during the year.
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Enjoy the summer!

Ms. Yaccarino

Name: _____ Calc BC-Summer Assignment

Section 1: Algebra Review

1. Solve: $xy + 2x + 1 = y$ *for y*

2. Factor: $x^2(x - 1) - 4(x - 1)$

3. Solve: $\ln(y - 1) - \ln x = x + \ln x$ *for y*

4. Factor: $8x^3 - 125$

Simplify each expression.

5. $\frac{(x^2)^3 x}{x^7}$

6. $\sqrt{x} \cdot \sqrt[3]{x} \cdot x^{\frac{1}{6}}$

7. $\frac{5(x+h)^2 - 5x^2}{h}$

8. $\frac{\frac{1}{x} + \frac{4}{x^2}}{3 - \frac{1}{x}}$

Simplify, by factoring first. Leave answers in factored form.

Example:

$$\frac{(x+1)^3(4x-9)-(16x+9)(x+1)^2}{(x-6)(x+1)} = \frac{(x+1)^2[(x+1)(4x-9)-(16x+9)]}{(x-6)(x+1)}$$

$$= \frac{(x+1)^2[4x^2-5x-9-16x-9]}{(x-6)(x+1)}$$

$$= \frac{(x+1)^2[4x^2-21x-18]}{(x-6)(x+1)}$$

$$= \frac{(x+1)^2[(4x+3)(x-6)]}{(x-6)(x+1)}$$

$$= (x+1)(4x+3)$$

9. $(x-1)^3(2x-3)-(2x+12)(x-1)^2$

10. $\frac{(x-1)^2(3x-1)-2(x-1)}{(x-1)^4}$

Simplify by rationalizing the *numerator.*

Example:

$$\frac{\sqrt{x+4}-2}{x} = \frac{\sqrt{x+4}-2}{x} \cdot \frac{\sqrt{x+4}+2}{\sqrt{x+4}+2} = \frac{x+4-4}{x\left(\sqrt{x+4}+2\right)} = \frac{x}{x\left(\sqrt{x+4}+2\right)} = \frac{1}{\sqrt{x+4}+2}$$

11. $\frac{\sqrt{x+9}-3}{x}$

12. $\frac{\sqrt{x+h}-\sqrt{x}}{h}$

Solve each equation or inequality for x over the set of real numbers.

13. $2x^4 + 3x^4 - 2x^2 = 0$

14. $\frac{2x-7}{x+1} = \frac{2x}{x+4}$

15. $\sqrt{x^2 - 9} = x - 1$

16. $|2x - 3| = 14$

17. $x^2 - 2x - 8 = 0$

18. $\frac{3x+5}{(x-1)(x^4+7)} = 0$

Solve each system algebraically.

19. $\begin{cases} x + y = 8 \\ 2x - y = 7 \end{cases}$

20. $\begin{cases} y = x^2 - 3x \\ y = 2x - 6 \end{cases}$

Section 2: Trigonometry Review

21. Use your knowledge of the unit circle, to evaluate each of the following. You MUST know your unit circle. Leave answers in radical form. Do NOT use your calculator.

a. $sin30°$

b. $cos\frac{2\pi}{3}$

c. $tan45°$

d. $sin\left(\frac{-\pi}{6}\right)$

e. $tan\pi$

f. $cos\frac{5\pi}{6}$

g. $\cos(90°)$

h. $cos\frac{3\pi}{4}$

i. $cot\frac{\pi}{6}$

j. $cos^{-1}\left(\frac{1}{2}\right)$

k. $sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

l. $tan^{-1}(1)$

Solve each trigonometric equation for $0 \leq x \leq 2\pi$.

22. $sinx = \frac{\sqrt{3}}{2}$

23. $tan^2x = 1$

24. $cos\frac{x}{2} = \frac{\sqrt{2}}{2}$

25. $2sin^2x + sinx - 1 = 0$

26. $3cosx + 3 = 2sin^2x$

Solve each exponential or logarithmic equation.

27. $5^x = 125$

28. $8^{x+1} = 16^x$

29. $81^{\frac{3}{4}} = x$

30. $8^{-\frac{2}{3}} = x$

31. $log_2 32 = x$

32. $log_x \frac{1}{9} = -2$

33. $log_4 x = 3$

34. $logx - \log(x-3) = 2$

Expand or condense each of the following use the laws of logs.

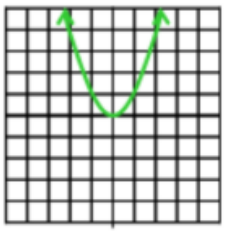
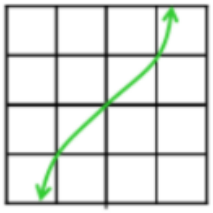
35. $log_3 5x^3$

36. $ln\frac{5x}{y^2}$

37. $2ln\sqrt{y} - \frac{1}{2}lny^4 + ln2y$

Section 3: Graphing Review

I. Symmetry – Even/Odd Functions

Quick Review		
Even Function	Symmetric about the y axis $f(-x) = f(x)$ for all x	Example: $y = x^2$ 
Odd Function	Symmetric about the origin (equivalent to a rotation of 180 degrees) $f(-x) = -f(x)$ for all x	Example: $y = x^3$ 

To determine algebraically if a function is even, odd, or neither, find *f*(-x) and determine if it is equal to *f*(x), -*f*(x), or neither.

Example: Determine if $f(x) = \frac{4x}{x^2+1}$ is even or odd.

$f(-x) = \frac{4(-x)}{(-x)^2+1} = \frac{-4x}{x^2+1} = -\frac{4x}{x^2+1} = -f(x)$ Therefore, $f(x)$ is an odd function.

Determine if the following functions are even, odd, or neither.

38. $f(x) = \frac{x^2}{x^4+3}$

39. $f(x) = \frac{x}{x+1}$

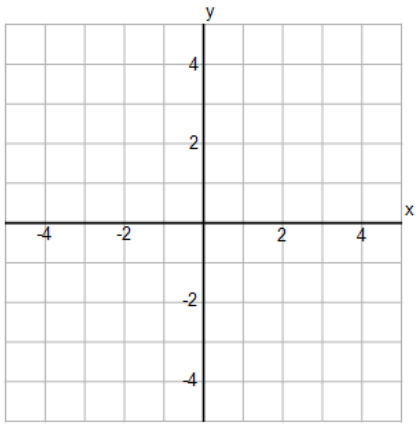
40. $f(x) = 1 + 3x^2 + 3x^4$

41. $f(x) = 1 + 3x^3 + 3x^5$

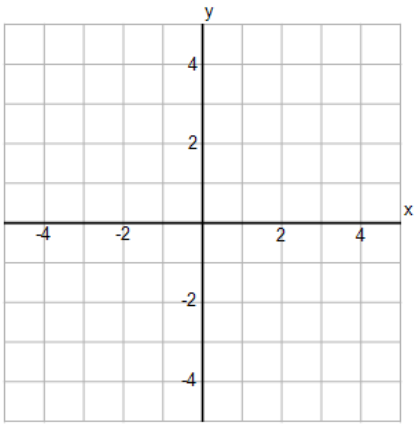
II. Essential Graphs

Sketch each graph. You should know the graphs of these functions.

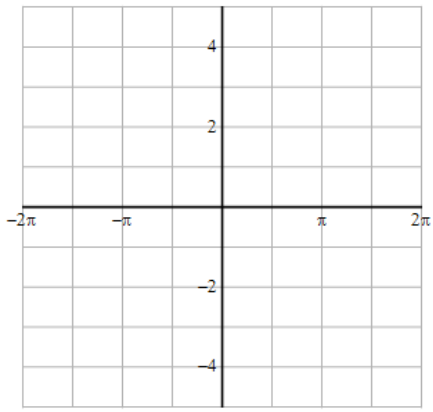
42. $f(x) = \sqrt{x}$



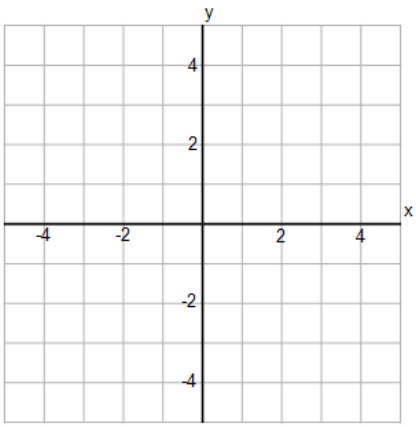
43. $f(x) = x^3$



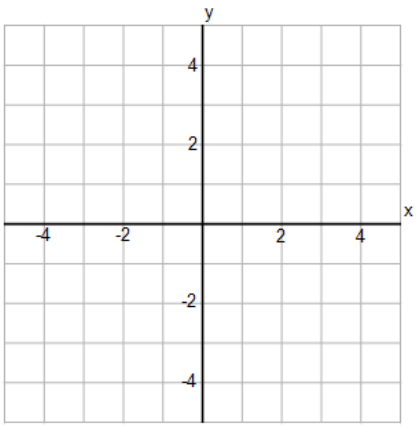
44. $f(x) = sinx$



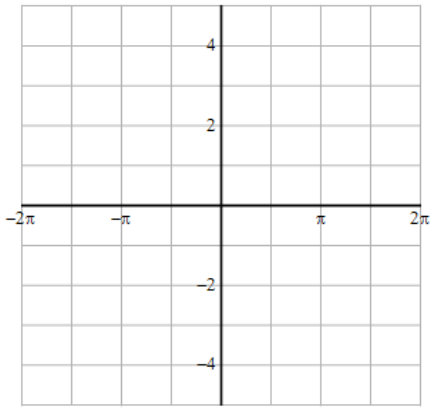
45. $f(x) = e^x$



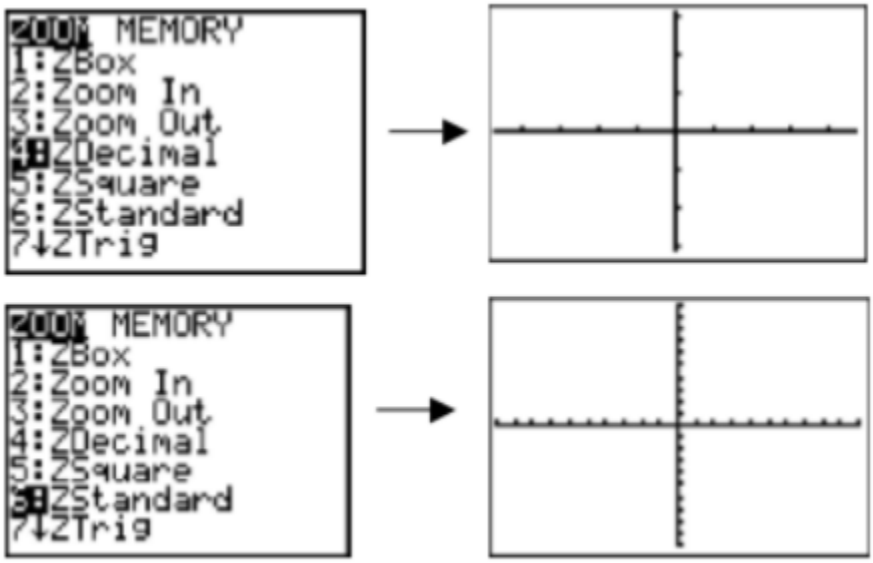
46. $f(x) = lnx$



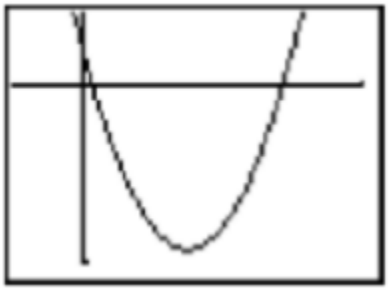
47. $f(x) = cosx$



Graphing Calculator Skill #1: You should be able to graph a function in a viewing window that shows the important features. You should be familiar with the built-in zoom options for setting the window such as zoom-decimal and zoom-standard. You should also be able to set the window conditions to values you choose.



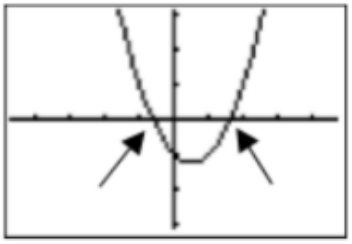
48. Find an appropriate viewing window to see the intercepts and the vertex defined by $y = x^2 - 11x + 10$. Use the window editor to enter the x and y-values.



Window: Xmin = _____
Xmax = _____
Xscl = _____
Ymin = _____
Ymax = _____
Yscl = _____

Graphing Calculator Skill #2: You should be able to graph a function in a viewing window that shows the x-intercepts (also called roots and zeros). You should be able to accurately estimate the x-intercepts to 3 decimal places. Use the built-in zero command on your graphing calculator.

49. Find the x-intercepts of $y = x^2 - x + 1$. *Window* [-4.7, 4.7] X [-3.1, 3.1]



(Write intercepts as points)

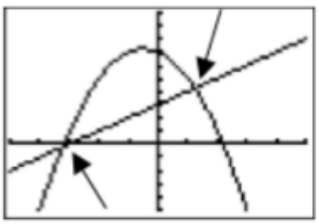
x-intercepts: _____

50. Find the x-intercepts of $y = x^3 - 2x + 1$. x-intercepts: _____

Graphing Calculator Skill #3: You should be able to graph two functions in a viewing window that shows the intersection points. Sometimes it is impossible to see all points of intersection in the same viewing window. You should be able accurately estimate the coordinates of the intersection points to 3 decimal places. Use the built-in intersection command.

51. Find the coordinates of the intersection points for the functions:

$$f(x) = x + 3 \text{ and } g(x) = -x^2 - x + 7$$



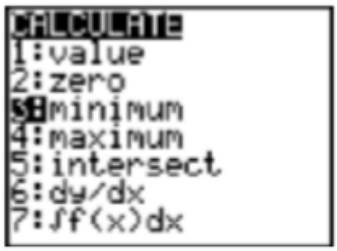
Intersection points: _____

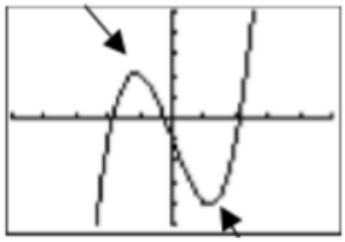
52. Find the coordinates of the intersection points of $f(x) = 4x^2$ and $g(x) = 2^x$.

Intersection points: _____

Graphing Calculator Skill #4: You should be able to graph a function and estimate the local maximum and minimum values to 3 decimals. Use the built-in max/min command.

53. Find the maximum and minimum values of the function $y = x^3 - 4x - 1$





(Value means the y-value)

Minimum value: _____

Maximum value: _____

54. Find the maximum and minimum values of the function $y = x^3 - 4x^2 + 4x$

55. Find the x-intercept(s), y-intercept, relative maximum and relative minimum of

$$y = x^3 + 2x^2 - 1$$

56. Find the coordinates of the points of intersection for

$$f(x) = 2x^2 + x - 9 \text{ and } g(x) = -\frac{3}{4}x + 3$$

Section 4: Linear Equations

57. Write the equation for the line in both forms given a slope and a point.

a. $m = \frac{2}{3}$ and *P*(3, 5)

b. $m = -\frac{4}{5}$ and *P*(1, 2)

Point-Slope: _____

Point-Slope: _____

Slope-Intercept: _____

Slope-Intercept: _____

58. Write the equation of the line passing through the given points in either point-slope or slope-intercept.

a. *P*(2, 2) and Q(4, 2)

b. *P*(3, -2) and Q(3, 7)

59. The slope of a line is $-\frac{1}{2}$ and the lines passes through the points (2, 5) and (-4, y). Find y.

Section 5: Average Rate of Change Review

Definition: Average Speed
Average speed is found by dividing the distance covered by the elapsed time.

$$\frac{\Delta y}{\Delta t} = \frac{\text{total distance traveled}}{\text{time elapsed}} = \frac{\text{final position} - \text{initial position}}{\text{final time} - \text{initial time}}$$

60. Find the average speed of a car that has traveled 350 miles in 7 hours.

61. Suppose $f(1) = 2$ and the average rate of change of f between 1 and 5 is 3. Find $f(5)$.

62. The position $p(t)$, in meters, of an object at time t, in seconds, along a line is given by $p(t) = 3t^2 + 1$.

a. Find the change in position between times t = 1 and t =4.

b. Find the average velocity of the object between times t = 1 and t = 4.

Section 6: Inverse Functions

63. Algebraically find the inverse of $y = \frac{3}{x-2} - 1$

64. If $f(x) = x^3 - 1$, find $f^{-1}(x)$ and verify that $f\left(f^{-1}(x)\right) = f^{-1}(f(x)) = x$

65. Discuss the relationship between the domain and range of a function and its inverse.

66. Given the one-to-one function f. The point (a, c) is on the graph of f. Give the coordinate of a point on the graph of f^{-1}.